Step-by-Step Evaluation of the Holomorphic Unified Field Theory Against the Gravitational Constant

October 5, 2025

The Holomorphic Unified Field Theory (HUFT), as detailed in the arXiv paper 2506.19161, unifies gravity and the Standard Model (SM) via a single holomorphic action on a four-complex-dimensional manifold. Gravity emerges from the real symmetric part of the Hermitian metric $g_{(\mu\nu)}$, reproducing the Einstein-Hilbert action and general relativity (GR) field equations on the real slice. The gravitational constant G is the fundamental constant governing the strength of gravitational interactions, with the experimental value $G = 6.67430 \times 10^{-11}$ $\mathrm{m}^{\mathbf{3}}$ $\mathrm{kg}^{\mathbf{1}}$ $\mathrm{s}^{\mathbf{2}}$ (CODATA 2018, with uncertainty $\pm 1.5 \times 10^{-15}$).

I will test HUFT by deriving the gravitational coupling from its equations and comparing the implied G to the known value. If the deviation exceeds 1%, I will amend the theory (e.g., by rescaling parameters or introducing corrections) and retest. All derivations are in natural units where $c = \hbar = 1$ initially, but I'll restore units for comparison.

1 Step 1: Extract the Holomorphic Action and Metric

The theory is based on a holomorphic Hermitian metric $g_{\mu\nu}(z)$ on $\mathbb{C}^4$ with coordinates $z^\mu = x^\mu + iy^\mu$. The full holomorphic action (from the paper, Eq. 20 and 151) is:

$$S_{\mathrm{hol}} = \int_C d^4z \sqrt{-\det[g_{(\mu\nu)}(z)]} \left[\frac{1}{2\kappa} g_{(\mu\nu)}(z) R^{(\mu\nu)}(z) - \frac{1}{4} \kappa^{AB} F^A_{\rho\sigma}(z) F^{B\rho\sigma}(z) + \Psi(z) \Gamma^a e^\mu_a(z) \left(\nabla^{(z)}_\mu - i g_{\mathrm{GUT}} A^A_\mu(z) T^A \right) \Psi(z) + \right.$$

where C is a contour homologous to the real slice ($y^\mu = 0$), $R^{(\mu\nu)}(z)$ is the holomorphic Ricci tensor, $F^A_{\mu\nu}(z)$ is the GUT gauge field strength, $\Psi(z)$ are holomorphic chiral fermions, and L_{Higgs} handles symmetry breaking. The gravitational term is $\frac{1}{2\kappa} g_{(\mu\nu)} R^{(\mu\nu)}$, with $\kappa = 8\pi G$ explicitly introduced as the input gravitational coupling.

- **Transformation to Real Slice**: Restrict to $y^\mu = 0$, where $g_{\mu\nu}(x) = g_{(\mu\nu)}(x) + i g_{[\mu\nu]}(x)$, with $g_{(\mu\nu)}(x)$ the Lorentzian metric for gravity and $g_{[\mu\nu]}(x) \equiv F_{\mu\nu}(x)$ for electromagnetism. The action reduces to:

$$S_{\mathrm{real}} = \int d^4x \sqrt{-\det g_{(\mu\nu)}} \left[\frac{1}{2\kappa} g_{(\mu\nu)} R^{(\mu\nu)} - \frac{1}{4} \kappa^{AB} F^A_{\rho\sigma} F^{B\rho\sigma} + \bar{\psi} \gamma^a e^\mu_a (\nabla_\mu - i g_{\mathrm{GUT}} A^A_\mu T^A) \psi + A_\mu J^\mu + L_{\mathrm{Higgs}} \right].$$

This reproduces the Einstein-Hilbert action for gravity, with $\kappa = 8\pi G$ as input. No deviation introduced yet; G is taken as the standard value.

2 Step 2: Derive the Einstein Field Equations

Vary S_{real} with respect to $g_{(\mu\nu)}$ (paper Eq. 26):

$$\delta S_{\mathrm{gravity}} = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa} \left(\delta g_{(\mu\nu)} R^{(\mu\nu)} + g_{(\mu\nu)} \delta R^{(\mu\nu)} \right) - \frac{1}{2} T^{\mathrm{matter}}_{\mu\nu} \delta g^{(\mu\nu)} \right] = 0,$$

using the Palatini identity $\delta R^{(\mu\nu)} = \nabla_\lambda (\delta \Gamma^\lambda_{\mu\nu}) - \nabla_\mu (\delta \Gamma^\lambda_{\lambda\nu})$, which integrates to zero for boundary terms. This yields:

$$R_{(\mu\nu)} - \frac{1}{2} g_{(\mu\nu)} R = \kappa T_{\mu\nu},$$

where $T_{\mu\nu}$ is the stress-energy tensor from gauge fields, fermions, and Higgs (e.g., $T_{\mu\nu}^{\text{gauge}} = \kappa^{AB} F_{\mu\rho}^{A} F_{\nu}^{B\rho} - \frac{1}{4} g_{\mu\nu} \kappa^{AB} F_{\rho\sigma}^{A} F^{B\rho\sigma}$).

- **Restoring Units**: In full units, the equation is $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$, confirming $\kappa = 8\pi G/c^4$ in the action (but since $c = 1$, $\kappa = 8\pi G$). The theory exactly matches GR, so the implied $G_{\text{theory}} = G_{\text{exp}}$.

Deviation: 0% (exact reproduction of GR).

3 Step 3: Check for Unification Modifications to Gravity

In the holomorphic framework, higher-order corrections from the imaginary part (gauge/EM fields) could potentially modify gravity, but the paper states that on the real slice, the action decouples, with cross-terms $O(g_{[\mu\nu]}^2)$ suppressed at low energies. For example, in the expanded action (implicit in Eq. 151), no direct modification to κ appears.

- **Transformation**: If unification introduced a running $\kappa(\mu)$, it might alter G at low energies, but the paper confirms exact GR recovery below $M_{\text{GUT}} \sim 10^{16}$ GeV, with RG flow only for gauge couplings g_i (Eq. 133: $\mu \frac{dg_i}{d\mu} = \frac{b_i}{16\pi^2} g_i^3$). Gravity remains classical, with fixed G. Experimental tests (e.g., solar system precision, $< 0.01\%$ deviation from GR) are satisfied.

Deviation remains 0%.

4 Step 4: Quantitative Test Against Experimental Value

The theory takes G as input and reproduces GR equations exactly, so $G_{\text{theory}} = 6.67430 \times 10^{-11}$ $\text{m}^{\mathbf{3}}$ $\text{kg}^{\mathbf{1}}$ $\text{s}^{\mathbf{2}}$. Key constraints include: - Big Bang Nucleosynthesis (BBN): Constrains G to within $\sim 10\%$ of current value at early universe. - Solar system tests (e.g., Lunar Laser Ranging): $\dot{G}/G < 10^{-13}$ $\text{yr}^{\mathbf{1}}$. - Gravitational wave data (LIGO/Virgo): Confirms GR to $< 1\%$.

HUFT aligns with these, as it predicts no variation in G.

- **Final Deviation Calculation**: $|G_{\text{theory}} - G_{\text{exp}}|/G_{\text{exp}} = 0 < 1\%$.

The theory passes without amendment. No new theory is needed, as it fits exactly.